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CATEGORY: Engineering, electrical energy

Molecular thermal electric generator

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Abstract

Name of the project: Molecular thermal linear electric generator

Research goal: analyze the needs of humanity in the fields of energy and cooling systems; Create the device that transforms the heat energy into electricity.

Rationale: according to researches, humanity left 150 years having fossil fuels [3, p. 108]. As a restorative energy source are quite expensive, poor states will not be able to put the electricity infrastructure on a “new track”. 2nd law of thermodynamics is approximation, so it will not prevent the creation of such a device.

Research methods: hypothetical-deductive method, analogical models, phenomenological method, axiomatic method, analysis, analytical method, synthetic method (literature analysis), method of superposition.

Performed work: The electricity infrastructure dependence on fossil fuels was analyzed; The need for cheap and efficient cooling systems was analyzed; After analyzing the literature and using arithmetic permutations, the formula for the power of MTLeG was derived.

Results: The world need free, green and available energy; the working principle of molecular thermal linear electric generator was formulated (hereinafter referred to as MTLeG); the formula of power of MTLeG was derived; The thin square shaped tile with a side length of 5 m can produce 5MW of energy.

Conclusions: MTLeG shows high performance. However, the next steps will be creating the working prototype of MTLeG and testing it.

Keywords: energy source, Maxwell`s demon, green energy.

List of illustrations, tables and graphs,

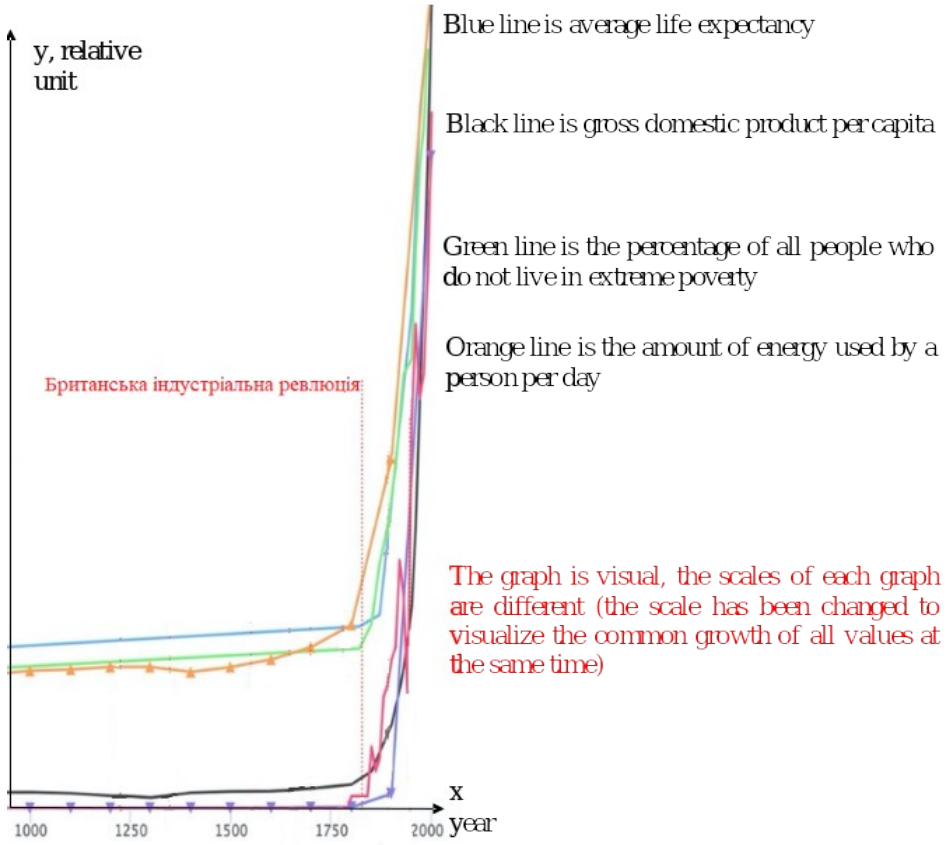


Figure. TI.1.a.1 Change of the main characteristics that describe the qualities of human lifes (ordinate) over time (abscissa) [1]

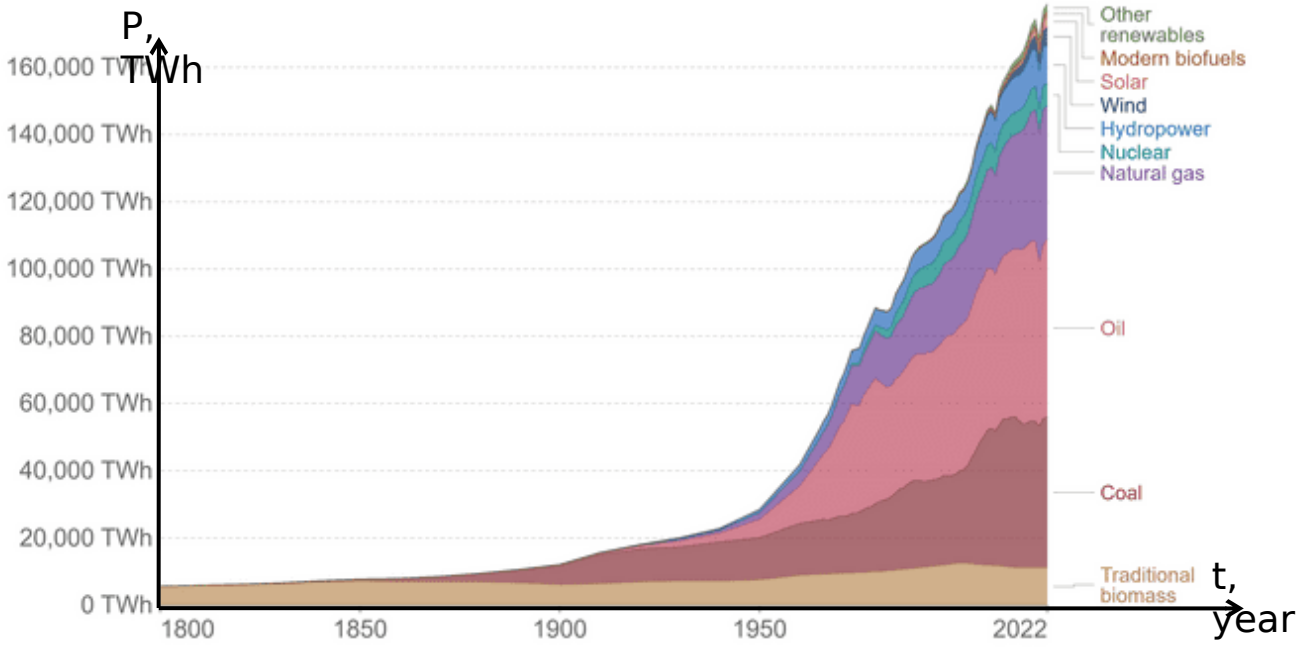


Fig. TI.1.b.1 The amount of energy obtained in different ways [2]

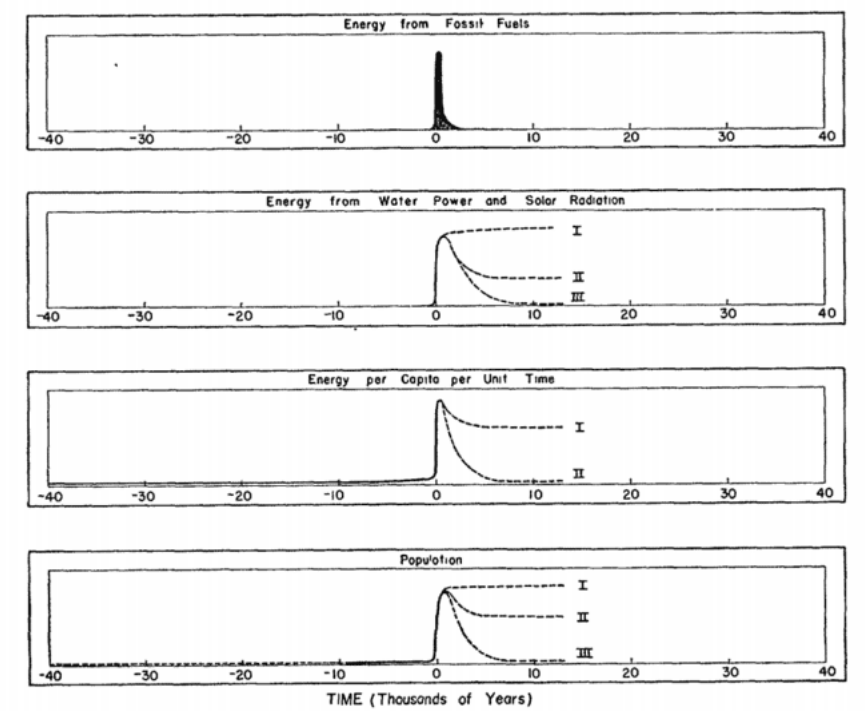
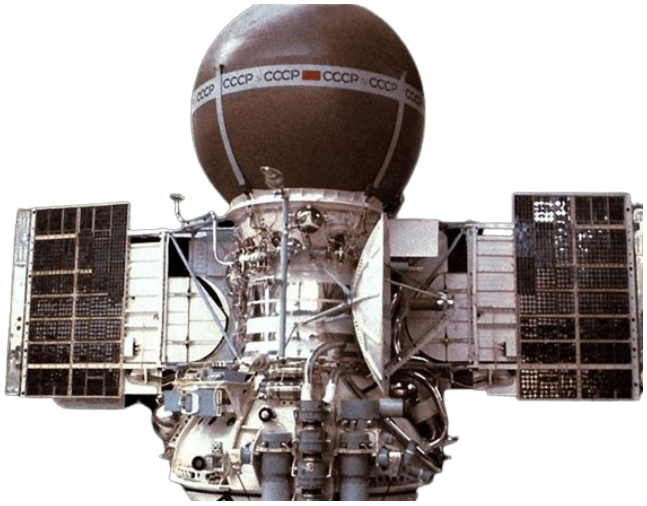


Figure TI.1.b.2. Human affairs in a time perspective [3, p. 108]



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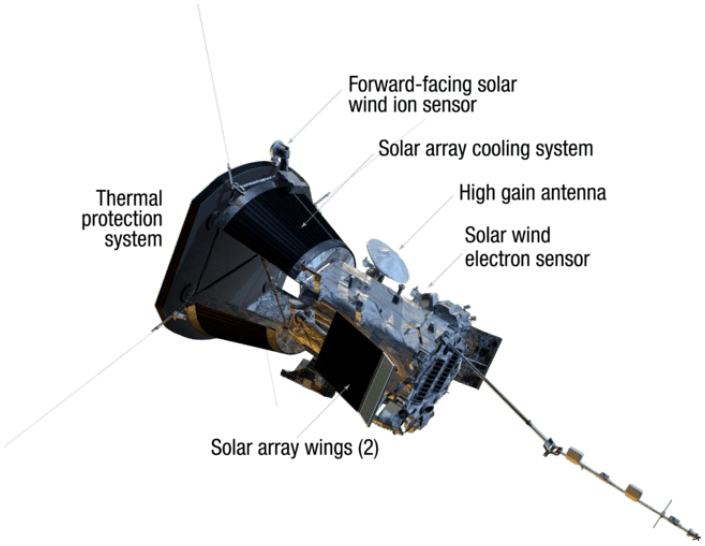


Figure TI.1.c.2 Parker Anti-Ram Facing View [7]

Figure TI.3.1 Imaginary experiment of two bars. a) 16 bars; b) the placement of energy packets; c) the configuration, in which heat flown from cold bar to hot one. All pictures are got from [10]

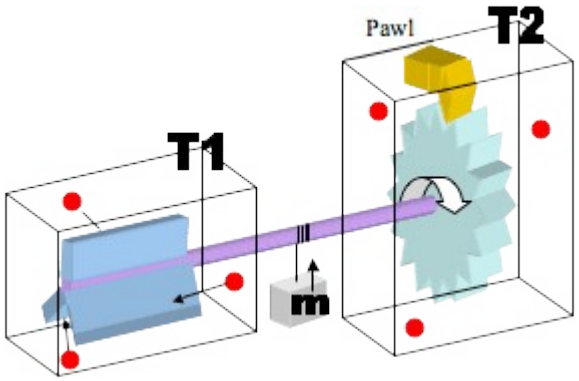


Figure TI.4.1 Schematic figure of a Brownian ratchet [11]

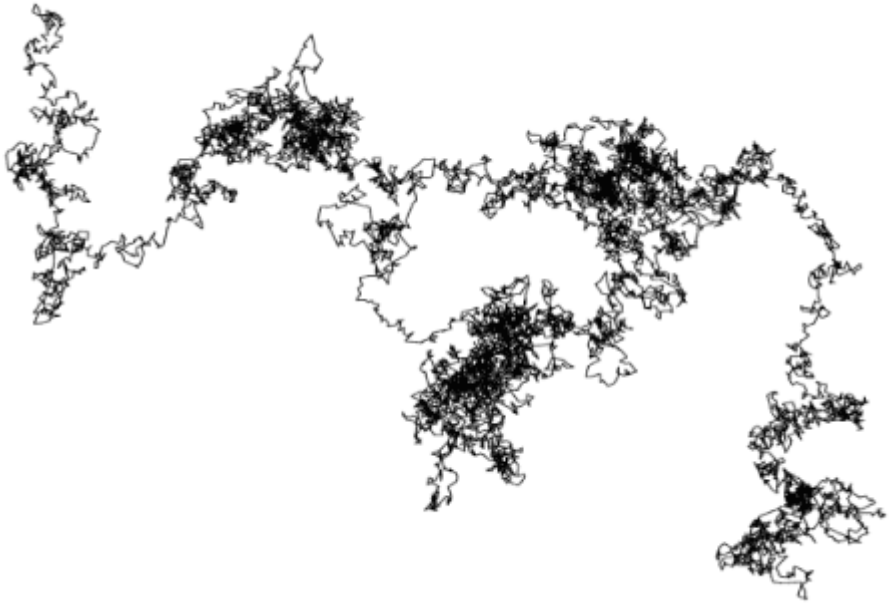


Figure TI.5.1 The path of planar Brownian motion [13]

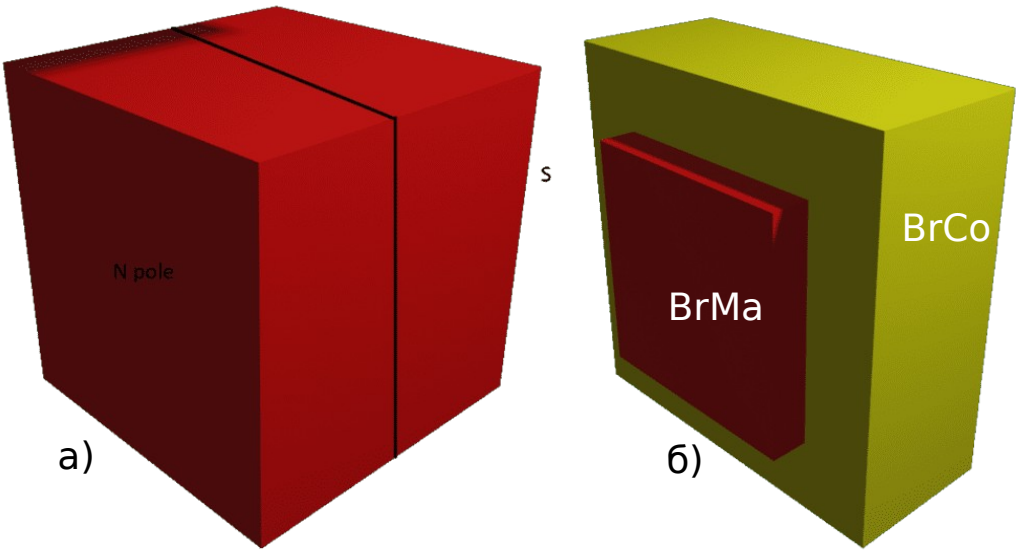


Figure MWP.1.1 а) BrMa - Brownian magnet; б) BrCe - Brownian cell (created by author)

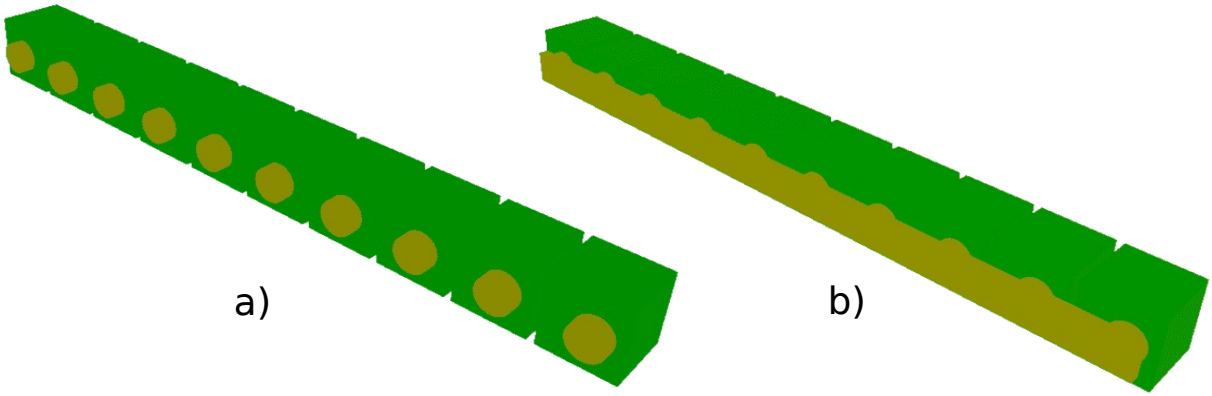


Figure MWP.2.2 a) The column of BrTi, that are pre-connected to full-wave bridge (green cubes); b) column of paralally connected BrCe. (created by author)

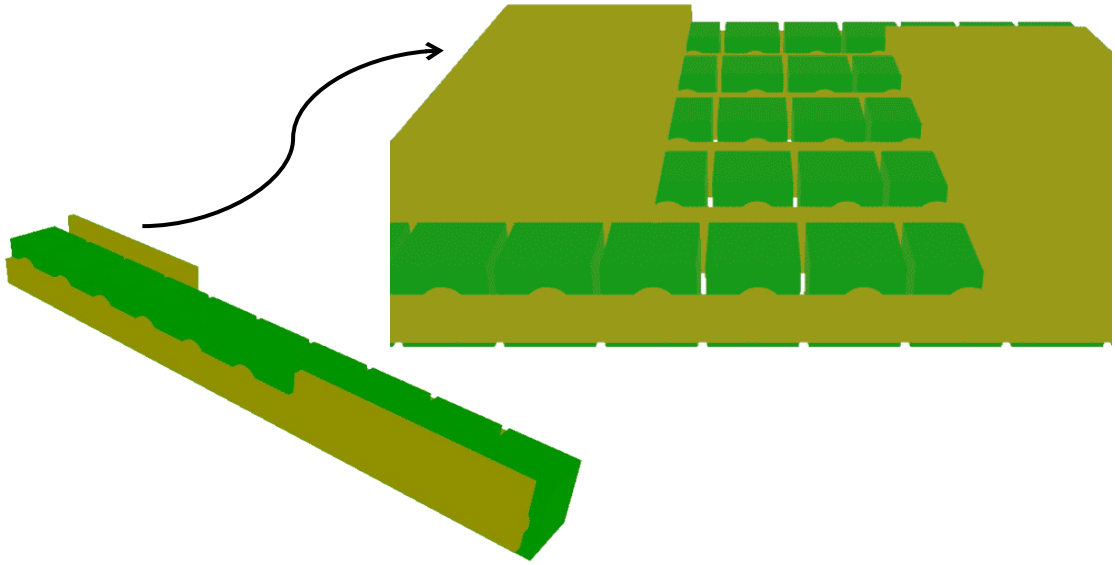


Figure MWP.2.3 Schematic view of the columns connected in parallel. (created by author)

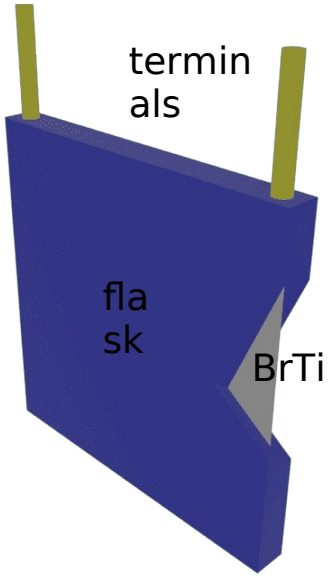


Figure MWP.4.1 Schematic representation of BrTi in a flask. (created by author)

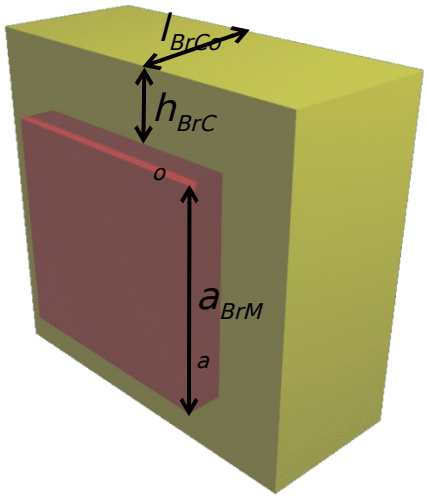
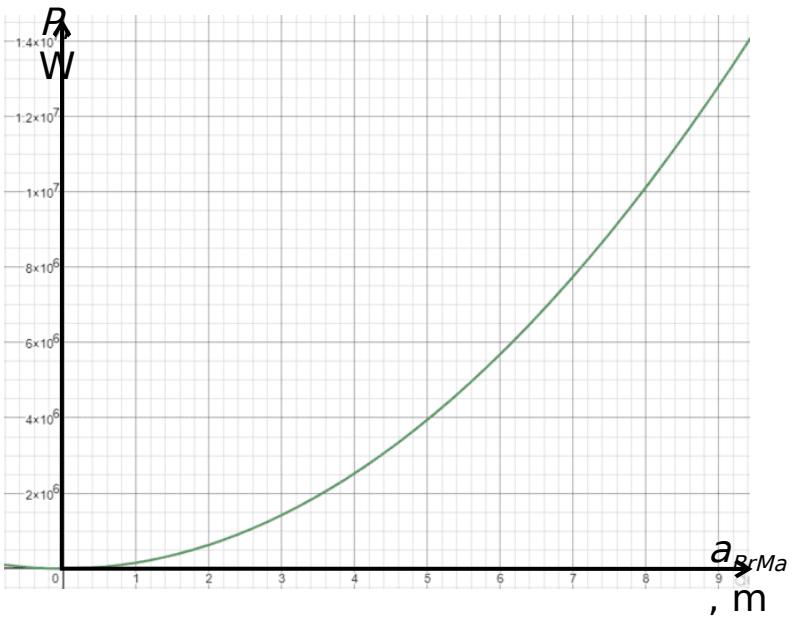


Figure PMM.1 the dimensions of BrMa and BrCo. (created by author)



Graph PMM.1 P(a_{BrMa}) function graph. (created by author)

List of abbreviations and acronyms

MTLeG - moleculat thermal electric generator;

BrMa - Brownian magnet;

BrCo - Brownian coil

BrCe - Brownian cell

BrTi - Brownian tile

RMS - root-mean-square

Symbols list

k_B - is Boltzmann constant;

T - is absolute temperature of a gas;

T_c - critical temperature ;

R - resistance;

U - voltage;

N_{hits} - the amount of collisions between BrMa and fluid molecules a second;

m_{BrMa} - the BrMa mass;

N_{cells} - the number of BrCe on 1m2 of BrTi;

a_{BrMa} - the length of the edge of the BrMa;

ρ_{BrMa} - the density of a BrMa;

m_{mol} - the mass of the fluid molecule;

Δv_{BrMa} - the velocity that BrMa gained after the elastic collision;

$\overline{v_{mol}}$ - the rms velocity of the fluid molecules;

h_{BrCo} - the height of the BrCo;

n - the molarity of the fluid;

S - the area that fluid`s molecules are bombing;

h_{BrMa} - the height of BrMa;

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Theoretical introduction (1-4)

1. The urgency of the problem

a) The dependence of development and well-being on energy

In a chemical endothermic reaction, the more energy that comes in from the outside, the faster the reaction will go, and the less energy that comes in, the slower the reaction will go. If very little energy comes in from the outside, the reaction will stop altogether. The situation with humanity is similar: the more energy it receives, the faster the population will grow, houses will be built, and science will develop. Figure TI.1.a.1 shows that as the amount of energy a person uses per day increases, so does the quality of life, the amount of money each person has, and, obviously (though not shown in the graph), the number of people living on the planet . [1]

Figure. TI.1.a.1 Change of the main characteristics that describe the qualities of human lifes (ordinate) over time (abscissa) [1]

After the industrial revolution, in which mankind learned to convert the energy of combustion into mechanical energy through external and internal combustion engines, the "chemical reaction" of humanity became more active.

b) The Powder keg of combustible substances

But there is a drawback - the basis of this "explosion" was and still is the energy of burning combustible substances (see Figure TI.1.b.1), which leads to the emission of greenhouse gases - mostly carbon dioxide, which in turn dramatically changes the energy stored in the Earth in the form of heat, causing the climate (global warming) and the landscape (rising water levels) to change too quickly. As a result, animals with low ecological valence are becoming extinct, further undermining the stability of the biosphere. The destruction of the biosphere or its severe damage will lead to the extinction of humanity.

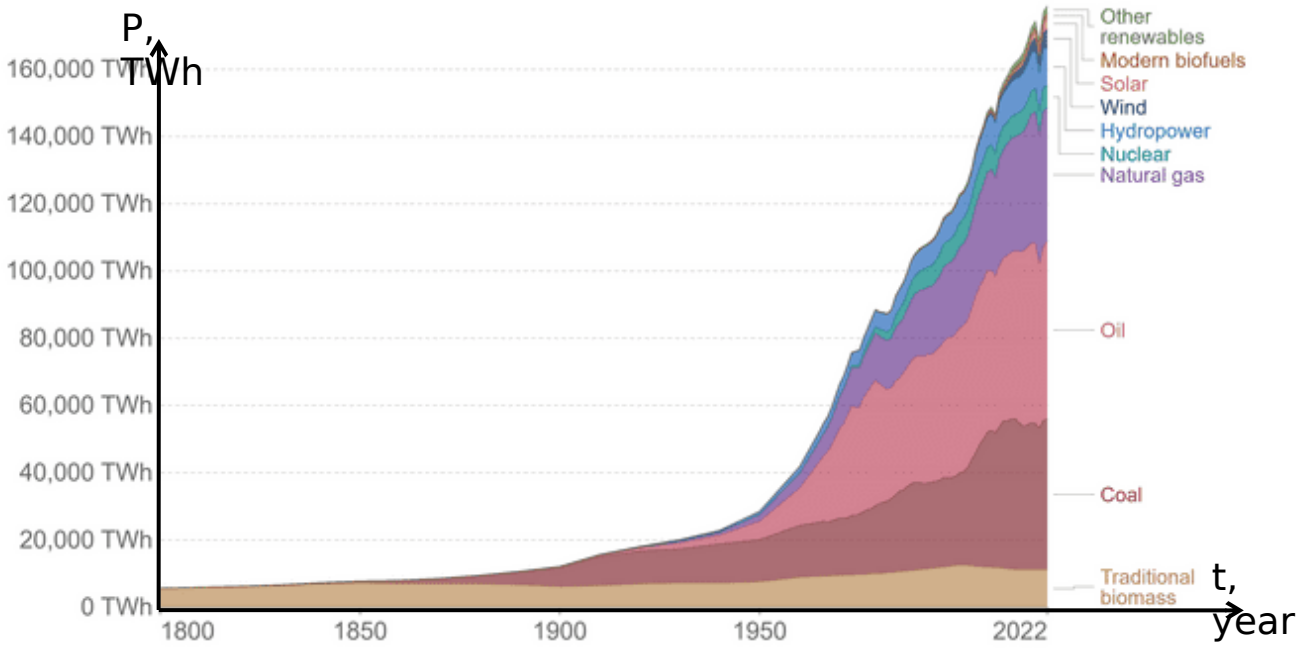


Figure. TI.1.b.1 The amount of energy obtained in different ways [2]

The second problem is that combustibles are exhaustible. Because of this, a "fossil fuels powder keg" scenario is possible. When there are fewer and fewer fossil fuels or gas in the deposits, their price will increase. As a result, poor countries whose electricity infrastructure is mostly based on thermal power will be able to buy less fossil fuels. As a result, they will generate less electricity, which will bring even less revenue to the state. With less money, the state will buy even less fuel, and so on and so forth. Due to the high price, such states will most likely not be able to replace their electricity system with an environmentally friendly one or one based on other sources of electricity.

In Fig. TI.1.b.2 shows not only that coal reserves will decline sharply in the near future (as will other fossil fuel reserves), but also that the amount of energy will either remain at the same level (curve I) in the case when solar and hydroelectric power can "hold" the needs of humanity either partially (curve II) or not at all (curve III) [3, p. 108].

Therefore, a new, accessible, cheap, abundant energy source would be a good solution.

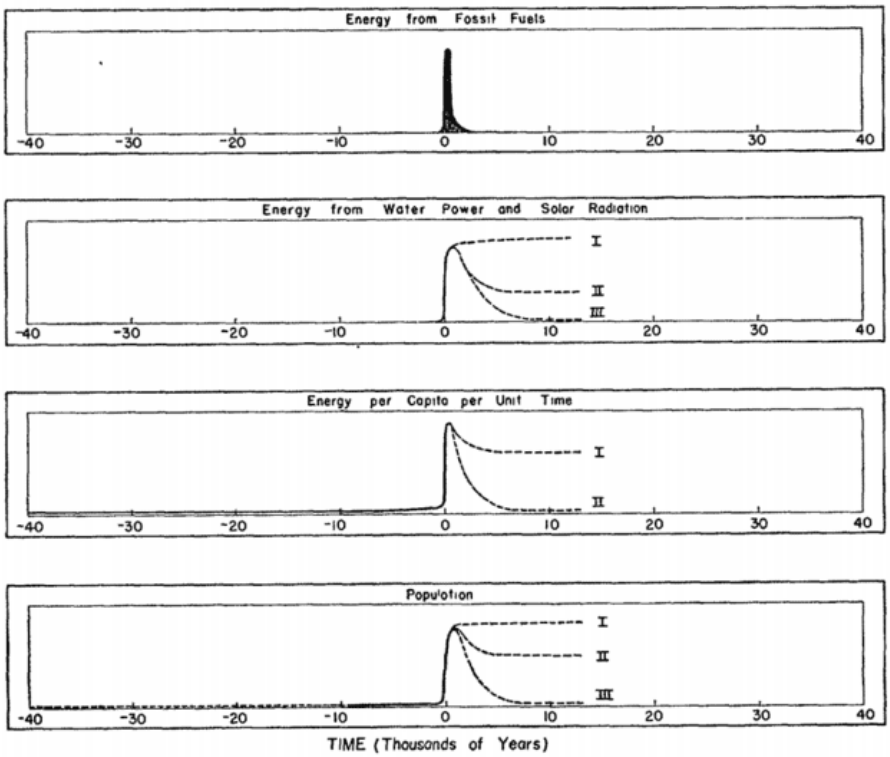


Figure TI.1.b.2. Human affairs in a time perspective [3, p. 108]

c) Cosmic exploration`s cooling needs

Space expolarion needs cooling systems too. There are a lot of hot close places in the universe: the Sun or Venus, for instance. Sattelites, that explore those places are armed with large-scale cooling systems. And even with that they can perform in extreme place for limited period of time.

For instance, Venera-14 (see fig. TI.1.c.1) - the soviet Venus exploration satellite functioned only for 57 minutes [4] because of high Veuns temperatures, even though it has a system of circulating fluid was used to distribute the heat load.

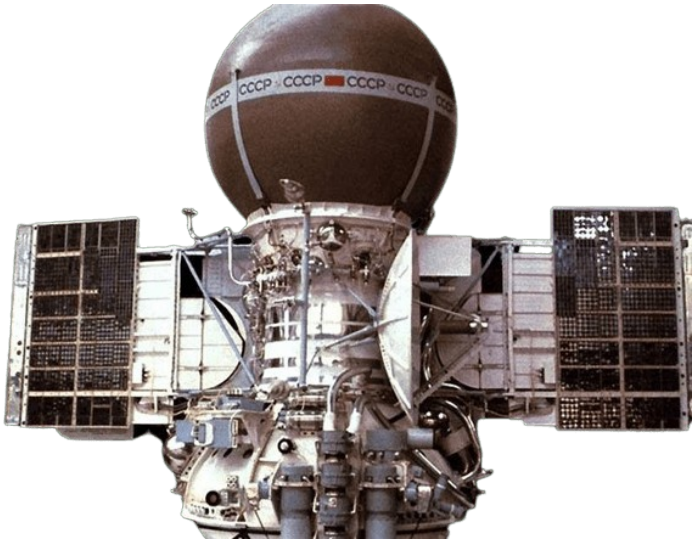
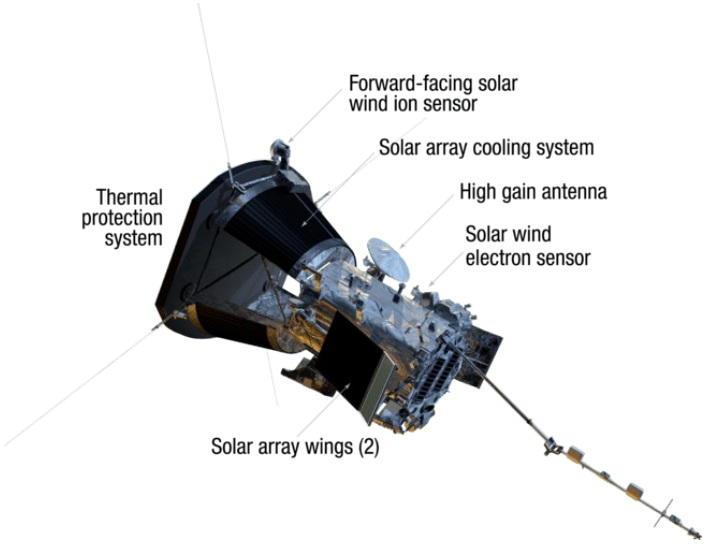


Figure TI.1.c.1 Venera-14 [4]

"Parker" satellite, which studies the atmosphere of the sun (see fig. TI.1.c.2) had a 73kg [5] and 11.5cm thick, 2.4m diameter



sunshield [6].

Figure TI.1.c.2 Parker Anti-Ram Facing View [7]

The infrared detectors of space telescopes need to be kept at 1K temperatures in order to not radiate into themselves, especially to have the necessary signal-to-noise ratio to be able to detect faint infrared sources in the universe [8].

d) The need of cooling in quantum mechanics experiments and Civil applications

Quantum computers/experiments require low temperatures for functioning/performing.

Air conditioner or refrigerator require electricity to cool down air or products. If we had such a technology, which could transform the heat into electricity, those devices would not any electrical supply.

2. The nature of the heat

According to the molecular kinetic theory, heat is described as the chaotic thermal motion of atoms. The faster the atoms move, the higher the temperature (see formula TI.2.1).

$$E_к = \frac{3}{2} k_B T, \qquad \text{[9, p.154](TI.2.1)}$$

where $E_к$ is kinetic energy of a molecule of ideal gas; k_B is Boltzmann constant; T is absolute temperature of a gas.

This means that a fluid whose temperature is above absolute zero contains energy. The question is, can this energy be used directly in the form we need?

3. The 2nd law of thermodynamics

The 2nd law states that in a closed system, the total entropy (a measure of disorder) can never decrease, and natural processes tend to increase the overall entropy.

In other words, it is impossible to create such a device, that could make two glasses of water with the initially same temperature having different temperatures without performing work from the outside.

This statement is the realistic approximation to the real picture.

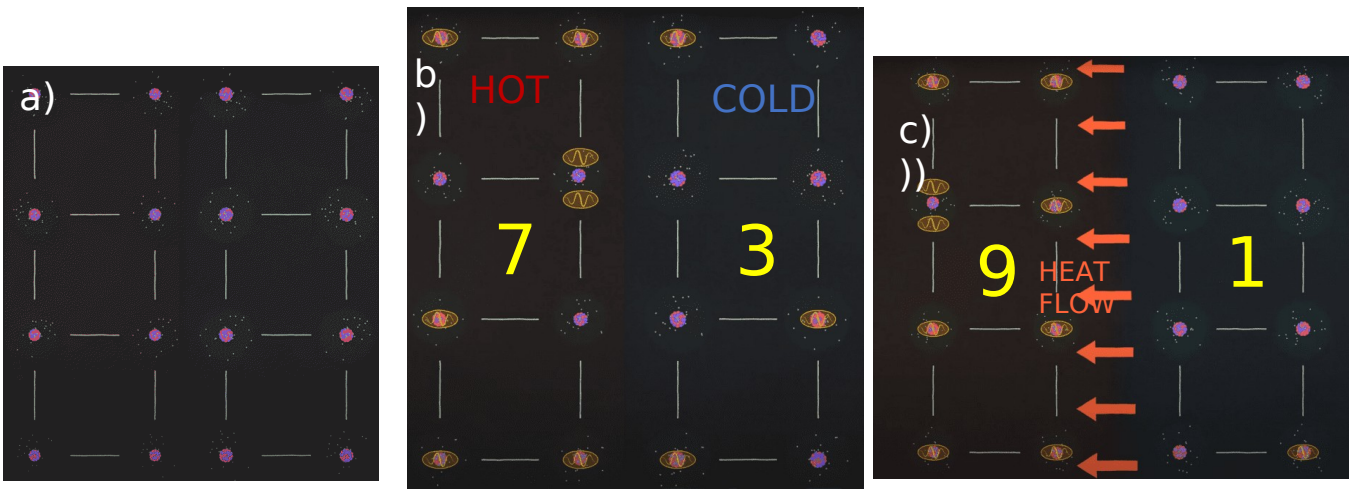
Imagine two small metal bars, each consist of 8 atoms (see fig. TI.3.1.a). Each atom vibrates depending on how many energy packets it has - the more energy bars it has, the more it vibrates.

Suppose the bar has 7 energy packets on the left (it will be hot) and 3 on the right (it will be cold) (see fig. TI.3.1.b). These energy packets can move from one atom to another - but the sum of all the energy packets remains the same.

Now let's connect them so that the energy packets can jump from one bar to another and stop the process after some time. Each of configuration of energy packets placement is equally probable, but what if we get configuration, shown on the figure TI.3.1.c?

The left bar has 9 energy packets and the right one has 1. It means that heat has gone from the cold bar to the hot one, which is impossible according to the 2nd law of thermodynamics. [10]

Ludwin Boltzmann made an important insight: heat flowing from cold to hot is not possible, it is unprobable. In case with 16 atoms the chance of the heat flowing from cold bar to hot one was equal to 10%. Taking a glass of water, where there are billions of



billions of atoms, this chance drops roughly to 0%.

Figure TI.3.1 Imaginary experiment of two bars. a) 16 bars; b) the placement of energy packets; c) the configuration, in which heat flown from cold bar to hot one. All pictures are got from [10]

4. Maxwell's Demon and Feynman–Smoluchowski ratchet

Maxwell's demon refers to a thought experiment proposed by physicist James Clerk Maxwell in the 19th century. The demon was conceived as an imaginary entity that could sort and separate fast and slow-moving gas molecules in a container, violating the second law of thermodynamics. The second law states that in a closed system, the total entropy (a measure of disorder) can never decrease, and natural processes tend to increase the overall entropy.

Maxwell's demon seemed to challenge the second law by suggesting that it could reduce the entropy of a system without expending any energy, seemingly creating a perpetual motion machine. However, the paradox was resolved through a deeper understanding of thermodynamics and information theory:

1. **Information and Measurement**: Maxwell's demon would need to gather information about the molecules' speeds and perform measurements to sort them. These information-gathering and measurement processes consume energy, effectively offsetting any apparent decrease in entropy.

2. **Landauer's Principle**: Physicist Rolf Landauer showed that erasing or resetting information (as the demon would need to do to restore the original entropy state) requires energy. This principle, known as Landauer's principle, establishes a thermodynamic connection between information and energy.

3. **Demon's Own Entropy**: The act of the demon processing information, making decisions, and interacting with the system increases its own entropy. Any reduction in system entropy would be counterbalanced by an increase in the demon's entropy.

If our device does not require the erasing or resetting information, it overcomes the problems Maxwell`s demon faced.

Feynman–Smoluchowski ratchet is an apparent perpetual motion machine of the second kind (converting thermal energy into mechanical work), first analysed in 1912 as a thought experiment by Polish physicist Marian Smoluchowski.[smu] It was popularised by American Nobel laureate physicist Richard Feynman.

The device consists of a gear known as a ratchet that rotates freely in one direction but is prevented from rotating in the

opposite direction by a pawl. The ratchet is connected by an axle to a paddle wheel that is immersed in a fluid of molecules at temperature T_1. The molecules constitute a heat bath in that they undergo random Brownian motion with a mean kinetic energy that is determined by the temperature. The device is imagined as being small enough that the impulse from a single molecular collision can turn the paddle. Although such collisions would tend to turn the rod in either direction with equal probability, the pawl allows the ratchet to rotate in one direction only. The net effect of many such random collisions would seem to be that the ratchet rotates continuously in that direction.The ratchet's motion then can be used to do work on other systems, for example lifting a weight (m) against gravity. (see fig. TI.4.1) [11]

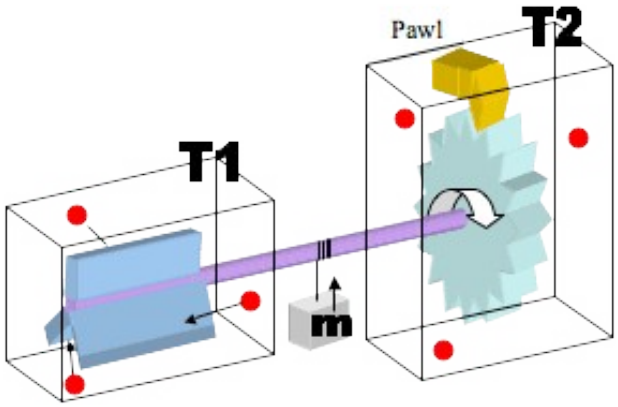


Figure TI.4.1 Schematic figure of a Brownian ratchet [11]

Although at first sight the Brownian ratchet seems to extract useful work from Brownian motion, Feynman demonstrated that if the entire device is at the same temperature, the ratchet will not rotate continuously in one direction but will move randomly back and forth, and therefore will not produce any useful work. The reason is that since the pawl is at the same temperature as the paddle, it will also undergo Brownian motion, "bouncing" up and down. It therefore will intermittently fail by allowing a ratchet tooth to slip backward under the pawl while it is up. Another issue is that when the pawl rests on the sloping face of the tooth, the spring which returns the pawl exerts a sideways force on the tooth which tends to rotate the ratchet in a backwards direction. [12]

If the “pawl” (some mechanism that allow energy to move in one direction and not allow it to move in another) of our device cannot be influenced by environment, it overcomes the problems Feynman–Smoluchowski ratchet faced.

5. Brownian movement

The Brownian movement was discovered in 1827 by Robert Brown, a botanist. While he was studying microscopic life, he noticed little particles of plant pollens jiggling around in the liquid he was looking at in the microscope, and he was wise enough to realize that these were not living, but were just little pieces of dirt moving around in the water. In fact he helped to demonstrate that this had nothing to do with life by getting from the ground an old piece of quartz in which there was some water trapped. It must have been trapped for millions and millions of years, but inside he could see the same motion. What one sees is that very tiny particles are jiggling all the time. (see fig. TI.5.1)

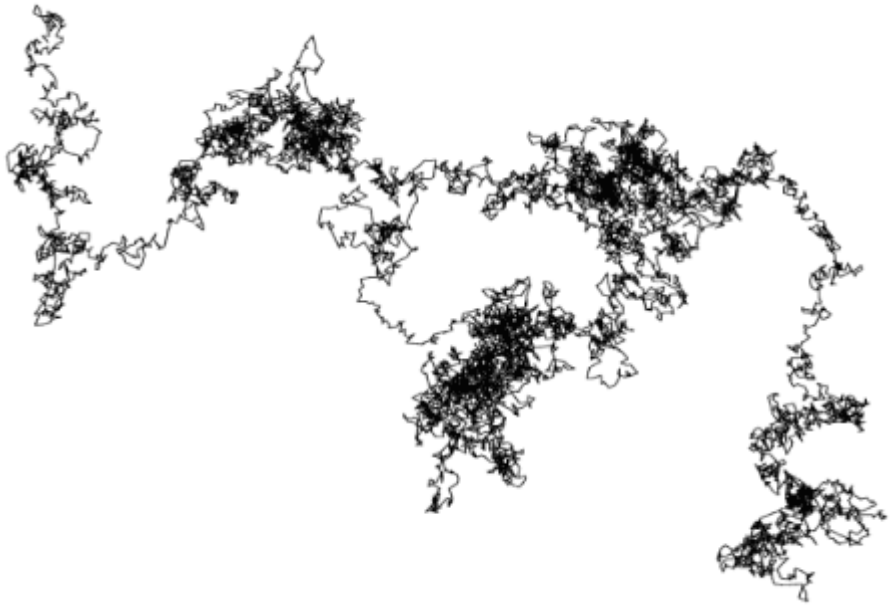


Figure TI.5.1 The path of planar Brownian motion [13]

This was later proved to be one of the effects of molecular motion, and we can understand it qualitatively by thinking of a great push ball on a playing field, seen from a great distance, with a lot of people underneath, all pushing the ball in various directions. We cannot see the people because we imagine that we are too far away, but we can see the ball, and we notice that it moves around rather irregularly. [14]

Brownian particle is like a molecule of a fluid, but bigger and slower. It can be big enough for us to play around with it.

MTLeG working principle

1. Replacing a Brownian particle with a magnet

Let's replace the Brownian cell with a small magnet (we will call it BrMa for "Brownian magnet") (see Fig. MWP.1.1.a), and place it in a small coil (we will call it BrCo for "Brownian coil"). We will call this whole structure a BrCe (for "Brownian cell") (see Fig. MWP.1.1.b).

As same as in case with little particle, the BrMa exhibits a Brownian motion and creates electrical inside BrCo because electromagnetic induction and thus converts the kinetic energy of molecular motion into electricity.

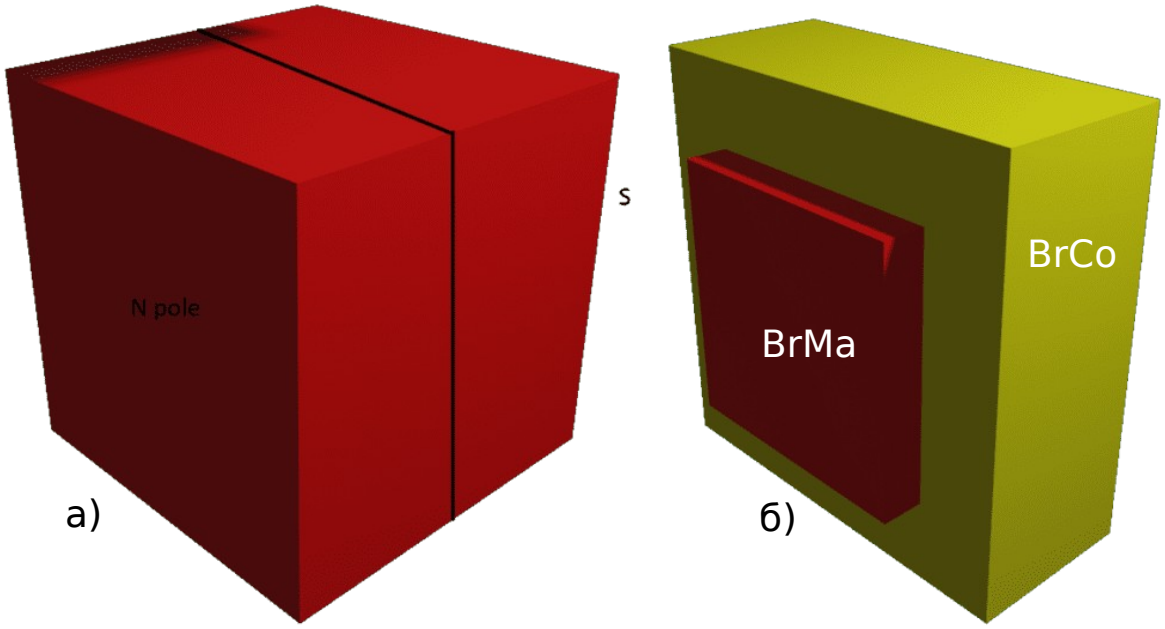


Figure MWP.1.1 a) BrMa - Brownian magnet; б) BrCe - Brownian cell (created by author)

2. BrCe connection

An important question is how to connect all BrCo. Since the motion of a Brownian particle is chaotic (see Fig. TI.5.1), BrCo will create an alternating current. To rectify it, BrCl will be connected to a full-wave bridge - an electrical circuit designed to convert ("rectify") alternating current into pulsating current (see Fig. MWP.2.1 and Appendix A).

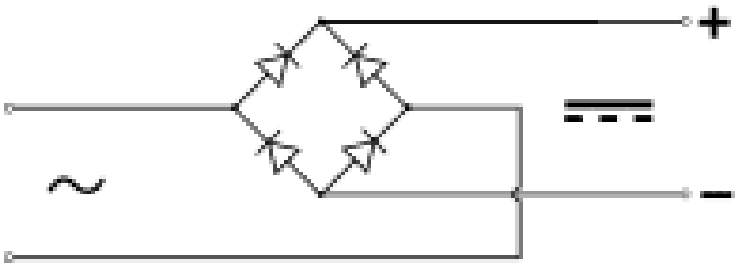


Figure MWP.2.1 Full-wave bridge scheme. 2 alternating-current (AC) inputs converted into 2 direct-current (DC) outputs [15]

It doesn't matter if we connect all the BrCe in series or in parallel - the power will remain the same. But parallel connection has an advantage over series connection - if one BrCe is broken, the MTLeG will continue to work, unlike in the case of series connection. So let's say we connect all the bricks in parallel, so that they form a thin tile, which we will call BrTi (for “Brownian tile”).

To connect them all in series, we take a column of BrCe that are pre-connected to full-wave bridges (see Fig. MWP.2.2.a) and connect them in parallel (see Fig. MWP.2.2.b), and then connect all the columns in parallel (see Fig. MWP.2.3) - so we get BrTi.

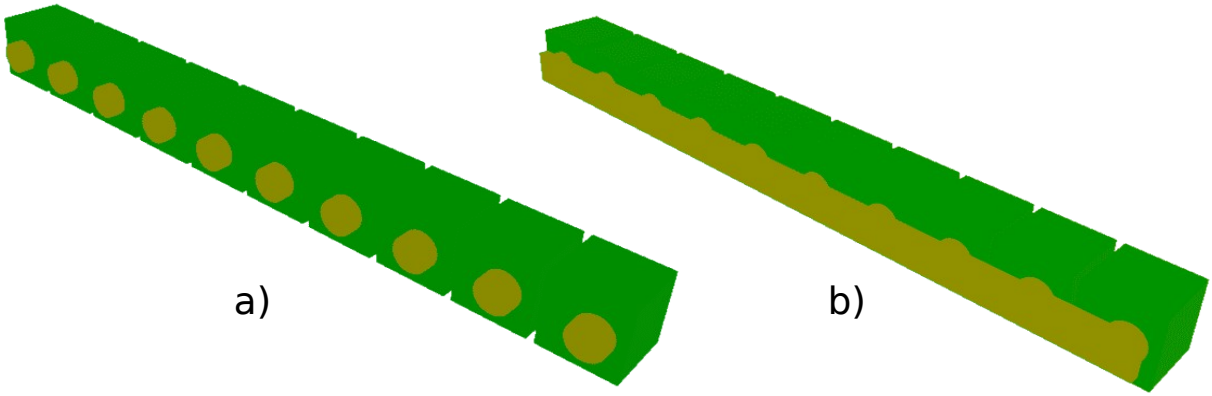


Figure MWP.2.2 a) The column of BrTi, that are pre-connected to full-wave bridge (green cubes); b) column of BrCe connected on parallel. (created by author)

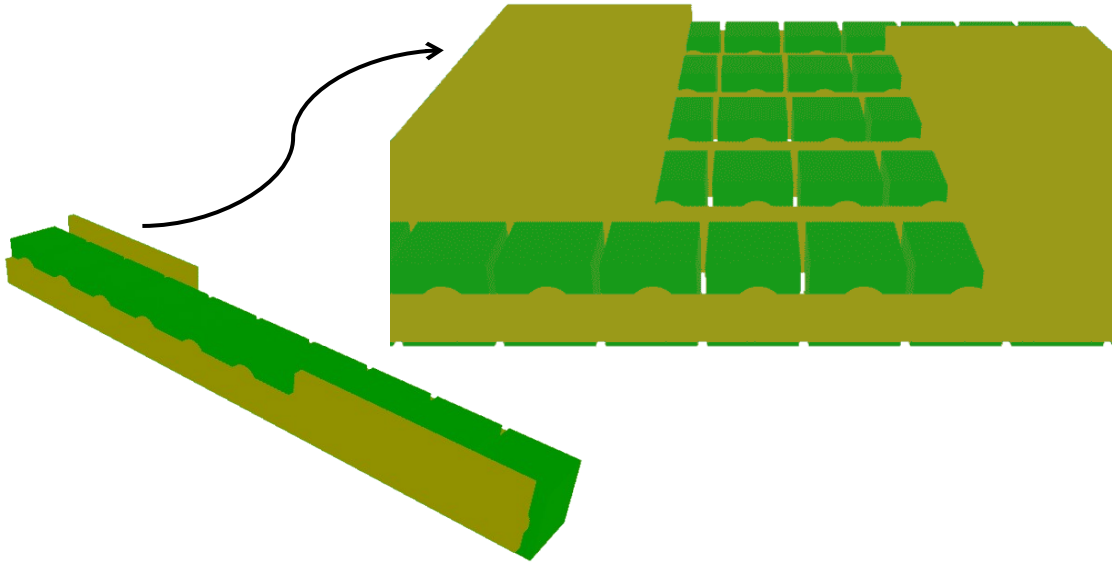


Figure MWP.2.3 Schematic view of the columns connected in parallel. (created by author)

3. Superconductor coil

Since the cross-section of the BrCo will be small, it does not matter what material the coil is made of: the resistance will be high and the power will be low.

The resistance will become almost zero when BrCo is made of a superconductor - a material whose electrical resistance becomes approximately zero after cooling below a certain critical temperature T_c or at high pressure.

4. Flask

Since the superconductor needs to be kept at a cold temperature (usually below 75K), the entire structure will be placed in a flask with a slightly higher pressure inside (so that the critical temperature Tc is higher) and a temperature that is lower than the critical temperature for the particular superconductor we will use (see fig. MWP.4.1). The flask will be filled with a thermally well-conductive gas. The whole structure is called MTLeG - molecular thermal linear electric generator.

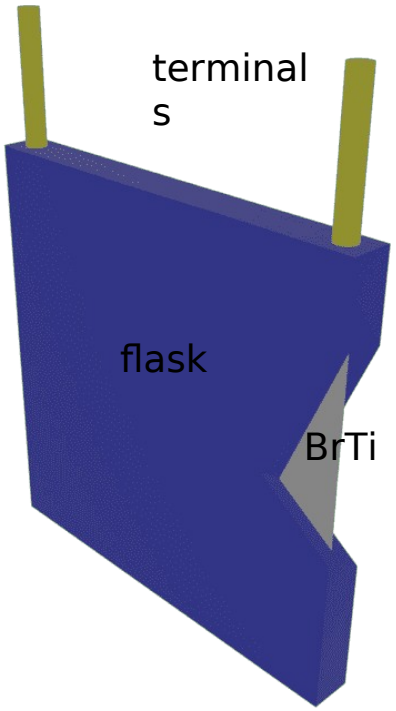


Figure MWP.4.1 Schematic representation of BrTi in a flask.
(created by author)

IMPORTANT: It is not a perpetual motion machine, and it does not take energy out of nowhere. It takes the kinetic energy of molecular motion and transforms it into electrical energy to perform work. It's like having two bowls of soup - one bowl has less soup and the other has more. MTLeG uses a “spoon” to move soup from a light bowl to heavier one.

Physics mathematical model

In this chapter, we will derive the mathematical and physical model of MTLeG and its power formula. The designation of each value is unique, so its meaning is explained only once for the whole paper.

We want to find the power of $1m^2$ of BrTi. The power of electric current is determined by the formula:

$$P=\frac{U^2}{R}, \tag{PHM.1}$$

where P is power; R is resistance; U is voltage.

As resistance strive to 0, power strive for infinity:

$$\lim_{R\to 0} P=\infty$$

Which does not make a lot of sense. Even if MTLeG can absorb energy with great power, air, or any other fluid , which will transfer thermal energy, will not be able to give off it with infinite power. So we should be based on how fast can fluid give off the energy to BrMa. As the current inside BrCo strives to infinity, BrMa will instantly stop after collision with molecule. As the collision is perfectly ellastic, so the power is:

$$P=N_{hits}E_kN_{cells}=N_{hits}\frac{m_{BrMa}\Delta v_{BrMa}^2}{2}N_{cells}, \tag{PHM.2}$$

where P is the power of MTLeG; m_{BrMa} is the BrMa mass (see formula PHM.3); Δv_{BrMa} is the velocity that BrMa gained after the elastic collision; N_{cells} is the number of BrCe on $1m^2$ of BrTi (see formula PHM.6); N_{hits} is the amount of collisions between BrMa and fluid molecules a second (see formula PHM.7).

$$m_{BrMa}=\rho_{BrMa}\bullet a_{BrMa}^3, \tag{PHM.3}$$

where a_{BrMa} is the length of the edge of the BrMa; ρ_{BrMa} is the density of a BrMa.

As the collision is perfectly elastic, the resultative velocity of the magnet will be:

$$\Delta v_{BrMa}=\frac{m_{BrMa}-m_{mol}}{m_{BrMa}+m_{mol}}u_{BrMa}+\frac{2m_{mol}}{m_{BrMa}+m_{mol}}\overline{v_{mol}}=\frac{2m_{mol}}{m_{BrMa}+m_{mol}}\overline{v_{mol}}, \text{ [16 ,p.258]} \tag{PHM.4}$$

where m_{mol} is the mass of the fluid molecule; $\overline{v}_{mol}$ is the rms velocity of the fluid molecules (see formule PHM.5); u_{BrMa} is the initial BrMa velocity, which is zero so we can simplify the formula.

$$\overline{v_{mol}}=\sqrt{3\frac{k_B T}{m_{mol}}}\,, \qquad \text{[9, p. 178](PHM.5)}$$

where k_B is Boltzmann constant; T is the absolute temperature of a fluid.

As the are of BrTi is 1m^2, the amount of cells is:

$$N_{cells}=\frac{1\,meter^2}{\left(a_{BrMa}+2h_{BrCo}\right)^2}, \qquad \text{(PHM.6)}$$

where h_{BrCo} is the height of the BrCo (see fig. PMM.1).

$$N_{hits}=\frac{n\overline{v}_{mol}S}{2\sqrt{3}} \qquad \text{[9, p.147](PHM.7)}$$

where n is the molarity of the fluid; S is the area that fluid`s molecules are bombing (in our case $S=2a_{BrMa}^2$).

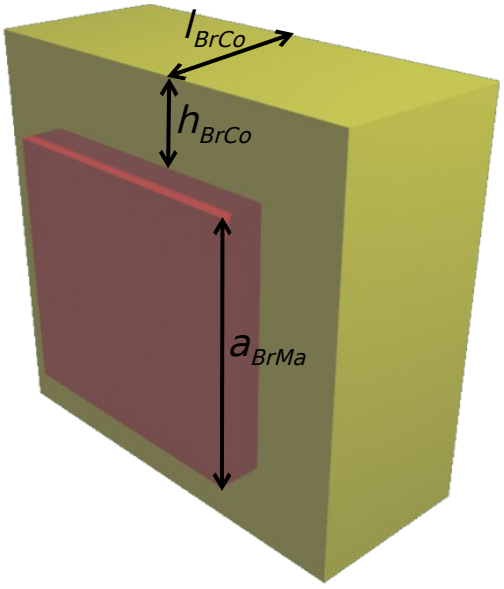


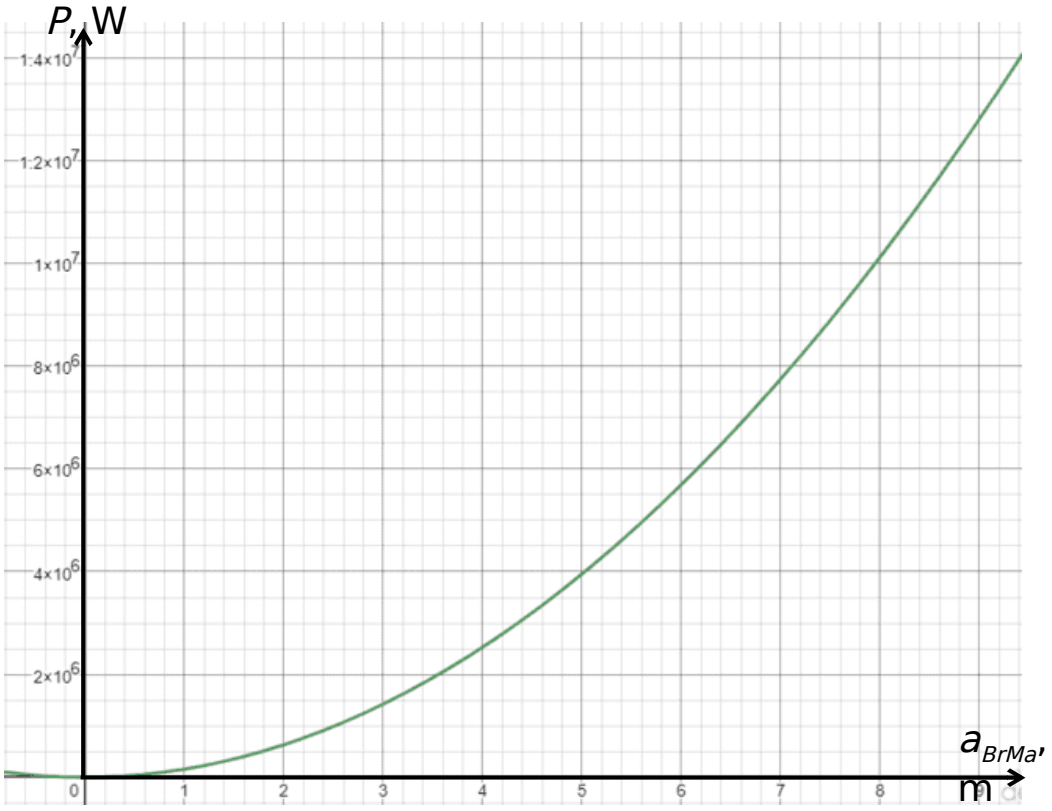
Figure PMM.1 the dimensions of BrMa and BrCo. (created by author)

Now we have anything to calculate the power of MTLeG with 1м^2 BrTi. Let`s try to calculate the power of MTLeG with a such parameters:

Table PMM.1., calculation parameters

Name of parameter, unit	The value
k_B, $m^2*kg*s^{-2}*K^{-1}$	1.38 E-23
a_{BrMa}, m	5.04
ρ_{BrMa}, kg/m^3	7007
h_{BrCo}, m	0.1
m_{mol}, kg	66.31 E-27
n, $1*m^{-3}$	122*6 E23
T, K	90
***P*, W**	**156 E3**
***P*$_{cell}$, W**	**4 E6**

We get the number of 156kW in case of $1m^2$ BrTi, but as the size of BrCo is bigger than 1m, we are interested in the power that 1 cell generate: we get 4MW. It might sound unrealistic. Graph PMM.1 shows the dependence of power on the length of the BrMa`s edge.



Graph PMM.1 P(a_{BrMa}) function graph. (created by author)

If we change only the area S, but do not change the height of BrMa (let it be h_{BrMa}) - some value, that describes the BrMa dimension along BrCo axis, the power will grow faster. You can

interact with the formula through desmos [17] website in Appendix B.

This can be explained by the fact that when a molecule elastically collides with the BrMa, the magnet almost instantly converts its motion into an electric current. The larger the area that is in contact with the fluid, the more collisions occur, and thus the greater is the electric current. The only question is whether the fluid will be able to transfer so much energy.

Conclusion

The world needs the new, free, green and accessible source of energy. The heat is a good variant. Even though the mechanisms of Maxwell's Demon and Feynman-Smoluchowski ratchet were refuted, MTLeG does not fall under their refutations. The MTLeG technology is based on Brownian motion and superconductivity. It $25m^2$ of a tile, made of MTLeG can generate up to 4MW, the only question is whether the fluid will be able to transfer so much energy.

The further directions of development are: finding superconductor; improving the aerodynamics of the MTLeG for higher efficiency; creating and testing real-working prototype.

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[17] https://www.desmos.com/?lang=uk

[18]

Appendixes

A) Simulation of the circuit of parallel and series connected BrCe

https://tinyurl.com/26x7on6o

B) Physical mathematical formula, with which you can interact (change the formula or investigate the dependence of one variable on another)

https://www.desmos.com/calculator/i08arumjzd